

1. $y = \frac{2}{3}(x-1)^{3/2}$ between $x=1$
and $x=2$

② length of this section.

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \left(\frac{2}{3}(x-1)^{3/2} \right) = \\ &= \frac{3}{2} \cdot \frac{2}{3} (x-1)^{3/2 - 1/2} \\ &= (x-1)^{1/2} \end{aligned}$$

$$\left(\frac{dy}{dx}\right)^2 = x-1 \quad S = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} = x^{1/2}$$

$$S = \int_1^2 x^{1/2} dx$$

OR

$$\begin{aligned} S &= \int_1^2 \sqrt{1 + \left((x-1)^{1/2}\right)^2} dx \\ &= \int_1^2 \sqrt{1 + (x-1)} dx \end{aligned}$$

AN

Substitute:

$$u = 1 + (x-1)$$

$$\frac{du}{dx} = 1$$

$$dx = du$$

$$x=1 \Rightarrow u=1$$

$$x=2 \Rightarrow u=2$$

$$\int_{u=1}^{u=2} \sqrt{u} du = \left[\frac{2}{3} u^{3/2} \right]_{u=1}^{u=2} = \frac{2}{3} \left[u^{3/2} \right]_{u=1}^{u=2}$$

$$= \frac{2}{3} \left[2^{3/2} - 1^{3/2} \right]$$

$$\begin{aligned} 8^{1/2} &= 4^{1/2} \times 2^{1/2} \\ &= 2 \times 2^{1/2} \end{aligned}$$

$$\begin{aligned} &\frac{2}{3} \left[(2 \times 2^{1/2}) - 1 \right] \\ &= \frac{4 \cdot 2^{1/2}}{3} - \frac{2}{3} \end{aligned}$$

$$2^{3/2} - (2^{1/2})^3 = (2^3)^{1/2} = 8^{1/2} = \frac{4}{3} \times 2^{1/2} - \frac{2}{3} = \underline{\underline{\frac{4\sqrt{2} - 2}{3}}}$$

$$\text{Volume } V = \pi \int_a^b y^2 dx$$

$$y = \frac{2}{3}(x-1)^{3/2}$$

$$y^2 = \frac{4}{9} \sqrt{(x-1)^3}$$

$$\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$$

$$V = \pi \int_1^2 y^2 dx$$

$$= \frac{4}{9} (x-1)^3$$

$$(a \cdot b)^n = a^n b^n$$

$$\frac{(2(x-1)^{3/2})^2}{3^2} = \frac{2^2 ((x-1)^{3/2})^2}{3^2}$$

$$= \frac{4(x-1)^3}{9}$$

$$V = \pi \int_1^2 \frac{4(x-1)^3}{9} dx$$

$$= \frac{4\pi}{9} \int_1^2 (x-1)^3 dx$$

Substitution

$$u = x-1$$

$$x=1 \Rightarrow u=0$$

$$x=2 \Rightarrow u=1$$

$$\frac{4\pi}{9} \int_0^1 u^3 dx$$

Power Rule

$$\int x^a dx = \frac{x^{a+1}}{a+1} + c$$

$$V = \frac{4\pi}{9} \int_1^2 (x-1)^3 dx$$

$$\frac{4\pi}{9} \cdot \frac{1}{4} [(x-1)^4]_1^2$$

$$\frac{4\pi}{9} \left[\frac{u^{3+1}}{3+1} \right]_0^1$$

$$\frac{4\pi}{9} \left[\frac{u^4}{4} \right]_0^1$$

$$V = \frac{\pi}{9} [(2-1)^4 - (1-1)^4]$$

$$\left[\frac{u^4}{4} \right]_0^1 = \frac{1}{4}$$

$$\frac{4\pi}{9} \cdot \frac{1}{4} = \frac{\pi}{9}$$

$$V = \frac{\pi}{9} [1^4 - 0^4]$$

$$V = \frac{\pi}{9}$$

$$\frac{\pi}{9} (1)$$

$$V = \frac{\pi}{9}$$

$$V = \frac{\pi}{9} [(2-1)^4 - (1-1)^4]$$

$$= \frac{\pi}{9}$$